VARIANCE ANALYSIS OF WOOL WOVEN FABRICS TENSILE STRENGTH USING ANCOVA MODEL

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Abstract: *The paper has conducted a study on the variation of tensile strength for four woven fabrics made from wool type yarns depending on fiber composition, warp and weft yarns tensile strength and technological density using ANCOVA regression model.*
In instances where surveyed groups may have a known history of responding to questions differently, rather than using the traditional sharing method to address those differences, analysis of covariance (ANCOVA) can be employed. ANCOVA shows the correlation between a dependent variable and the covariate independent variables and removes the variability from the dependent variable that can be accounted by the covariates.
The independent and dependent variable structures for Multiple Regression, factorial ANOVA and ANCOVA tests are similar. ANCOVA is differentiated from the other two in that it is used when the researcher wants to neutralize the effect of a continuous independent variable in the experiment.
The researcher may simply not be interested in the effect of a given independent variable when performing a study. Another situation where ANCOVA should be applied is when an independent variable has a strong correlation with the dependent variable, but does not interact with other independent variables in predicting the dependent variable's value. ANCOVA is used to neutralize the effect of the more powerful, non-interacting variable. Without this intervention measure, the effects of interacting independent variables can be clouded.

Keywords: *ANCOVA model, fiber composition, tensile strength.*

1. INTRODUCTION

Analysis of covariance models combines analysis of variance with techniques from regression analysis. With respect to the design, ANCOVA models explain the dependent variable by combining categorical (qualitative) independent variables with continuous (quantitative) variables. There are special extensions to ANCOVA calculations to estimate parameters for both categorical and continuous variables. ANCOVA models can, however, also be calculated using multiple regression analysis using a design matrix with a mix of dummy-coded qualitative and quantitative variables. In the latter approach, ANCOVA is considered as a special case of the General Linear Model (GLM) framework [1].

The study was conducted on woven materials made of combed wool type yarns used for manufacturing outwear clothing, on 63 articles structured as follows [2], [3]:

- Group A: 13 articles with $Nm_{warp}=Nm_{weft}$; 11 articles with $Nm_{warp}\neq Nm_{weft}$. Total: 24 articles;
- Group B: 13 articles with $Nm_{warp}=Nm_{weft}$; 4 articles with $Nm_{warp}\neq Nm_{weft}$. Total: 17 articles;
- Group C: 4 articles with $Nm_{warp}=Nm_{weft}$; 5 articles with $Nm_{warp}\neq Nm_{weft}$. Total: 9 articles;
- Group D: 13 articles with $Nm_{warp}=Nm_{weft}$. Total: 13 articles

The variation limits of the composition and structural characteristics for the tested woven materials are indicated in Table 1.

Table 1: Variation limits of composition and structural characteristics

Group/Fibrous composition		Nm_{Warp}	Nm_{Weft}	T_{warp} Twist $_{Warp}$ (twist/m)	T_{weft} $Twist_{Weft}$ (twist/m)
Group A 100%Wool	min	40/2	24/1	530	410
	max	64/2	37/1	740	730
Group B 45%Wool +55% PES	min	48/2	30/1	600	510
	max	64/2	64/2	780	740
Group C 44% Wool + 53%PES + 3% D	min	56/2	37/1	510	480
	max	60/2	60/2	730	730
Group D 60% PES + 40% Celo	min	52/2	52/2	420	400
	max	64/2	64/2	500	500

The tensile testing was performed using an H 1K-S UTM Tinius Olsen (Hounsfield) testing machine, with a 1 kN load cell. The tests were done accordingly to standard (SR EN ISO 2062, 2002), on both directions – weft and warp [4], [5], [6].

2. EXPERIMENTAL PART

2.1. Collection, systematization and processing of experimental data

Based on the experimental data, the following variables were included in the ANCOVA regression model:

- dependent variable (Y) is tensile strength variance, Pr(daN);
- nominal independent variable is group of woven fabrics depending on fiber composition (100% Wool, 100% PES, 45% Wool + 55% PES, 45%Wool + 52% PES + 3% Dorlastan) – three dummy variables D_1, D_2 respective D_3;
- quantitative independent variables (X1 respective X2) are warp yarns tensile strength P_{rwarp} (daN) and weft yarns technological density D_{weft} (yarns/10cm).

2.2. Hypothesis formulation

H_0: there are no significant differences between tensile strength values of woven fabrics depending of fiber composition, warp yarns tensile strength P_{warp} (N) and weft yarns technological density D_{weft} (yarns/10cm);

H_1: there are significant differences between tensile strength values of woven fabrics depending of fiber composition, warp yarns tensile strength P_{warp} (N) and weft yarns technological density D_{weft} (yarns/10cm), (H_0 is rejected).

2.3. Formulation of the regression model

Nominal independent variable "fiber composition" has four categories so will be three dummy variabels also called alternative variables. Reference woven fabrics (D_1, D_2, D_3) will be those from 100 % wool. Therefore, all the interpretation will be done compared to this group of woven fabrics.
The dummy variables are defined in table 2.

Table 2: *Definition of dummy variables*

Group	D_1	D_2	D_3	Fiber Composition
1	1	0	0	45% wool + 55% PES
2	0	1	0	45% wool+ 52% PES + 3% Dorlastan
3	0	0	1	100% PES
4	0	0	0	100% wool

The ANCOVA model with three dummy variables is defined as relation:

$$Y = a_0 + a_1 D_1 + a_2 D_2 + a_3 D_3 + b1\ X1 + b2\ X2 + \varepsilon \quad \textbf{(1)}$$

The regression, as a conditioned mean, has the following forms:

$$M (Y/D) = a_0 + b1\ X1 + b2\ X2 \quad D_1, D_2, D_3 = 0 \quad \textbf{(2)}$$

for tensile strength variance of 100% wool woven fabrics;

$$M (Y/D) = (a_0 + a_1) + b1\ X1 + b2\ X2 \quad D_1 = 1 \quad D_2, D_3 = 0 \quad \textbf{(3)}$$

for variance of tensile strength of 45% Wool + 55% PES woven fabrics;

$$M (Y/D) = (a_0 + a_2) + b1\ X1 + b2\ X2 \quad D_2 = 1 \quad D_1, D_3 = 0 \quad \textbf{(4)}$$

for tensile strength variance of 45% Wool + 52% PES + 3% Dorlastan woven fabrics;

$$M(Y/D) = (a_0 + a_3) + b1\ X1 + b2\ X2 \quad D_3 = 1 \quad D_1, D_2 = 0 \tag{5}$$

for tensile strength variance of 100% PES woven fabrics.

The coefficients of ANCOVA model defined in table 3 were determined for the established model. Estimators and model estimations are defined similar previous models.

Table 3: *ANCOVA model - coefficients*

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	38,890	12,651		3,074	0,003
	D_1	18,660	5,024	0,160	3,714	0,000
	D_2	-17,705	4,697	-0,211	-3,769	0,000
	D_3	73,901	10,835	0,673	6,820	0,000
	D_{weft} (yarns/10cm)	2,414E-02	,070	0,044	0,344	0,732
	Pr_{warp}(N)	3,207	1,436	0,203	2,233	0,029

a Dependent Variable: Pr_{fabric}(daN)

The estimations of model are: a_0 = 38,890; a_1 = 18,660; a_2 = -17,705; a_3 = 73,901; b_1 = 0,024; b_2 = 3,207

The estimated ANCOVA model has the following expression:

$$Y = 38{,}890 + 18{,}660\ D_1 - 17{,}705\ D_2 + 73{,}901\ D_3 + 0{,}02414\ X1 + 3{,}207\ X2 \tag{6}$$

2.4. Model interpretation

The model interpretation is the following:

a) a_0 = **38,890** is the mean value estimate tensile strength of the fabrics from 100 % wool while the tensile strength of the warp yarns and the weft yarn density technology may be encoded to 0 (X1 = 0, X2 = 0).

b) $a_0 + a_1$ = 38,890 + 18,660 = **57,55** is the mean value estimate tensile strength of woven fabrics from 45 % wool + 55 % PES while the tensile strength of the warp yarns and the weft yarn density technology may be encoded to 0 (X1 = 0, X2 = 0).

c) $a_0 + a_2$ = 38,890 – 17,705 = **21,185** is the mean value estimate tensile strength of woven fabrics from 45 % wool + 52 % PES + 3 % Dorlastan while the tensile strength of the warp yarns and the weft yarn density technology may be encoded to 0 (X1 = 0, X2 = 0).

d) $a_0 + a_3$ = 38,890 + 73,901 = **112,791** is the mean value estimate tensile strength of woven fabrics from 100 % PES while the tensile strength of the warp yarns and the weft yarn density technology may be encoded to 0 (X1 = 0, X2 = 0).

e) b_1 and b_2 shows the tensile strength variance of fabrics type wool while while the tensile strength of the warp yarns and the weft yarn density technology increases by one unit.

It is observed from the table of coefficients that sig. is <0.05 (except sig. D_{weft} (yarns/10cm), the value being 0.732)

Note: Although the sig. D_{weft} (yarns/10cm) exceeds 0.05 (which shows that the weft yarn density does not significantly influence the dependent variable) was not excluded Backward processing method (see Table 4)

Table 4: *Variables Entered/Removed*

Model	Variables Entered	Variables Removed	Method
1	Pr_{warp}(N), D1, D2, D3, Db(fire/10cm)	0,000	Enter

a All requested variables entered.
b Dependent Variable: Pr_{fabric}(daN)

Interpreting the sig value can conclude that there are significant differences between the values of tensile fabric depending on fiber composition, tensile strength of warp yarns Pr_{warp} (N) (the hypothesis Ho it is rejected).

Technological density of weft yarns D_{weft} (yarns/10cm) does not significantly influence the variation of tensile strength of 100% wool woven fabrics.
-Ho: between the values of tensile fabric no significant differences based on fiber composition, tensile strength of warp yarns Pr_{warp} (N) and technological weft density D_{weft} (yarns/10cm)
-H1: between the values of tensile fabric there is significant differences depending on fiber composition, tensile strength of warp yarns Pr_{warp}(N) and technological weft density D_{weft} (yarns/10cm)

2.5. Hypothesis confirmation over errors

2.5.1. M(ε) = 0 (errors mean is nule)

Hypothesis:

- Ho: M (ε) = 0
- H1: M (ε) ≠ 0

The Student t-test t for error (Unstandardized Residual) evaluation is applied as shown in table 4.

Table 4: *Student t-test for testing of errors mean*

	Test Value = 0					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
Unstandardized Residual	0,000	63	1,000	-2,1094237E-15	-1,3212957	1,3212957

(Sig.= 1 > 0,05), so hypothesis H_0 is accepted, errors mean is zero.

2.5.2. V (εi) = σ^2 homoscedasticity hypothesis)

A non-parametric correlation test is applied between the estimated errors and the dependent variable (D_1, D_2, D_3).The correlation coefficient Spearman is calculated and the Student t-test for this coefficient is performed. The results are indicated in table 5.

Hypothesis:

- H_0: the correlation coefficient is insignificantly larger than zero (the null hypothesis of the Student t-test is accepted);
- H_1: the correlation coefficient is significantly larger than zero (the null hypothesis of the Student t-test is rejected).

The values of sig. for the correlations D_1 – estimated errors (0,840), D_2 – estimated errors (0,976), D_3 – estimated errors (0,355), D_{weft} (yarns/10cm) – estimated errors (0,091), Prwarp (N) - estimated errors (0,114) are bigger than 0.05, which means that the null hypothesis of the Student test is rejected, so the model is homoscedastic (see Table5).

2.5.3. εi ~ N (0, σ2) – normality hypothesis

Testing the normality of error distribution is done with the nonparametric Kolmogorov-Smirnov test (see Table 5).

Table 5: *Testing of the normality hypothesis. One-Sample Kolmogorov-Smirnov Test*

		Unstandardized Residual
N		64
Normal Parameters	Mean	-1,9790605E-09
	Std. Deviation	5,2895718
Most Extreme Differences	Absolute	0,051
	Positive	0,039
	Negative	-0,051
Kolmogorov-Smirnov Z		0,406
Asymp. Sig. (2-tailed)		0,997

a Test distribution is Normal.
b Calculated from data.

Table 6: *Spearman test for verifying the homoscedasticity hypothesis*

			D_1	D_2	D_3	D_{weft} (yarns/10cm)	Pr_{warp} (N)	Unstandardized Residual
Spearman 's rho	D_1	Correlation Coefficient	1,000	-0,202	-0,132	0,389	0,403	-0,026
		Sig. (2-tailed)	0,000	0,109	0,297	0,001	0,001	0,840
		N	64	64	64	64	64	64
	D_2	Correlation Coefficient	-0,202	1,000	-0,218	-0,750	-0,751	-0,004
		Sig. (2-tailed)	0,109	0,000	0,083	0,000	0,000	0,976
		N	64	64	64	64	64	64
	D_3	Correlation Coefficient	-0,132	-0,218	1,000	0,573	0,561	0,118
		Sig. (2-tailed)	0,297	0,083	0,000	0,000	0,000	0,355
		N	64	64	64	64	64	64
	D_{weft} (yarns/ 10cm)	Correlation Coefficient	0,389	-0,750	0,573	1,000	0,975	0,213
		Sig. (2-tailed)	0,001	0,000	0,000	0,000	0,000	0,091
		N	64	64	64	64	64	64
	Pr_{warp}(N)	Correlation Coefficient	0,403	-0,751	0,561	0,975	1,000	0,199
		Sig. (2-tailed)	0,001	0,000	0,000	0,000	0,000	0,114
		N	64	64	64	64	64	64
	Unstandardized Residual	Correlation Coefficient	-0,026	-0,004	0,118	0,213	0,199	1,000
		Sig. (2-tailed)	0,840	0,976	0,355	0,091	0,114	0,000
		N	64	64	64	64	64	64

** Correlation is significant at the 0.01 level (2-tailed).

The Sig = 0,997 (higher than 0,05), therefore the normality hypothesis H_0 is accepted.

2.5.4. cov (εi, εi) – testing of errors autocorrelation

Hypothesis:

- H_0: $\rho = 0$ (the errors are not auto-correlated);
- H_1: $\rho \neq 0$ (the errors are auto-correlated).

The verification is done with the Durbin Watson test and the results are shown in Table 7.

Table 7: *Durbin Watson test for errors auto-correlated testing*
Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	0,989	0,979	0,977	5,51	1,649

a Predictors: (Constant), Pr_{warp}(N), D_1, D_2, D_3, D_{weft}(yarns/10cm)
b Dependent Variable: Pr_{fabric}(daN)

The value of 1.649 is compared with the calculated value of test (dl, du). It has been observed that the obtained value is contained in the (du, 4 – du) interval. Therefore, the nule hypothesis is accepted (the recorded errors are auto-correlated).

2.6. Testing collinearity of independent variables

Table 8: *Testing collinearity of independent variables*
Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	38,890	12,651		3,074	0,003		
	D_1	18,660	5,024	0,160	3,714	0,000	0,193	5,177
	D_2	-17,705	4,697	-0,211	-3,769	0,000	0,115	8,712
	D_3	73,901	10,835	0,673	6,820	0,000	0,037	27,041
	D_{weft} (yarns/10cm)	2,414E-02	0,070	0,044	0,344	0,732	0,022	45,113
	Pr_{warp}(N)	3,207	1,436	0,203	2,233	0,029	0,043	23,016

a Dependent Variable: Pr_{fabric}(daN)

The indicator VIF has high value ranging between (5.177, 45.113), which indicates that there is collinearity between the independent variables used in the model.

3. CONCLUSIONS

a) The ANCOVA model permits us to evaluate the homogeneous character of a population by separating and testing of the effects caused by the considered factors.

b) If the test data obtained after trials on the elements of a sample taken from one homogeneous population are divided in distinct groups, then the mean values of the groups do not differ significantly between themselves.

c) In the case of a non-homogeneous population, the deviations of the individual values as compare to the mean value are not anymore accidental and when dividing the test data in distinct groups the mean values are differ between themselves significantly because of some causes having a systematic action.

d) ANCOVA model included as independent variables, both dummy or alternative variables (fiber composition) and numerical variables (tensile strength of warp and weft yarn density technology) and as the dependent variable, tensile strength woven fabrics.

e) By applying and interpreting regression model ANCOVA sig value can conclude that there are significant differences between the variance of tensile strength of woven fabrics depending on fiber composition, tensile strength of warp yarns Pr_{warp} (N) (the null hypothesis Ho is rejected).

f) Technological weft density D_{weft} (yarns/10cm) does not significantly influence the tensile strength variation of woollen woven fabrics.

REFERENCES

[1] Hristian L., Bordeianu D. L. and Lupu I. G., *"The quantitative and qualitative analysis of woven fabrics type wool surface characteristic using Anova model"*, Annals of the university of Oradea Fascicle of Textiles, Leatherwork, vol. VIX, 2013, No. 2, pp. 50-56.

[2] Hristian L., *„Substantion and Elaboration of a New Testing Methodology of Woven Materials Surface Characteristics with Classical Testing Methods*", Ph.D. Thesis, "Gheorghe Asachi" Technical University, Jassy, 2008.

[3] Hristian L., Bordeianu D. L. and. Lupu I. G, "*The quantitative and qualitative analysis of woven fabrics type wool surface characteristics using Anova model",* Annals of the University of Oradea, Fascicle of Textiles, Leatherwork, vol. VIX, 2013, No. 2, pp. 50-56.

[4] Hristian L., Lupu I. G. and Cramariuc O., "*Employing the ANOVA model for the differentiation of wool type woven materials based on their handle properties",* International Scientific Conference 19-20 November, Gabrovo,Bulgaria, 2010, vol. II,

[5] Neculaiasa M. S., Hristian L., "*Textile metrology*," vol. I, Ed. Performantica, 2004, Iasi.

[6] *Standards SR EN ISO 13934-1:2002-Textiles-Tensile properties of fabrics−Determination of maximum force and elongation at maximum force using the strip method.*